THEORY OF DRY FRICTION OF RUBBERY MATERIALS

Yu. A. Buevich and A. I. Leonov

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Friction of solids involves short-range forces between adjacent surface layers, which are largely determined by the shape and structure of those layers, which are themselves determined to a considerable extent by the relative velocity. A theory of friction thus involves the microstructure and the detailed physical phenomena near the surfaces.

However, most existing theories are based on phenomenological (essentially macroscopic) concepts (see [1] for a survey), though the explicit use of microscopic concepts is presented in [2], where it is shown that one elastic body sliding over another gives rise to elastic waves that carry energy away from the contact surface. This loss may be treated formally as due to a tangential force resisting the motion. The force defined in this way has a falling velocity characteristic.

There is much evidence that the friction differs greatly from that for ordinary elastic bodies if one body (or both) should be highly elastic (rubber, polymer, etc) [3]. A model describing these differences would be of considerable interest.

Here we consider the somewhat idealized case of a rubbery body sliding over a crystalline one; the frictional force is deduced as a function of the velocity and other parameters. The surfaces are taken as smooth and clean, while the bodies are homogeneous. Various simplifying assumptions are made, but these are unimportant from the qualitative standpoint.

1. PHYSICAL MODEL OF FRICTION

Consider the motion (relative velocity v) of a rubberlike body 1 in the space $z > 0$ (Fig. 1a) over the elastic body z in the space $z < 0$. The bodies interact via discrete sets of force centers distributed throughout the volumes; the detailed nature of these centers will not be discussed, but they may be supposed to be microroughness or (if the gap δ is very small) individual groups of chain molecules. The only important point is that the interaction occurs via this set of centers.

1. Body 2 has a completely periodic structure; the force centers (points in Fig. 1a) are joined by rigid bonds. Possible oscillation and displacement of these centers are neglected. The repeat distance 2L of body 2 in the direction of the x axis is then independent of the velocity.

2. The surface layer of body 1 consists of sawtooth chains packed in a direction parallel to the motion, these chains consisting of force centers of separation a. These chains can be straightened by the tensions produced by the motion. Then the surface of body 1 has a periodic structure, the repeat distances being $2l$ along the x axis and 2b along the z axis (Fig. 1). Real chains have a spatial structure, and their links may lie in a variety of directions; here we simplify the model by assuming that the chains lie in planes parallel to the (x, z) plane. It will become clear that the qualitative results are not affected.

3. Relative motion produces tensile stresses that increase with the frictional force Q. These forces cause the chains to straighten by reduction of the angle α, distance a (Fig. 1b) remaining unaltered. These chains thus resemble the long spiral molecules or fibers of a rubbery body, while the change in α is due to the elastic forces. Here $l = l(Q)$, and

$$dl / dQ > 0, \qquad l(0) = l_0 . \tag{1.1}$$

We also also assume that there is a finite limit l_∞ as $Q \to \infty$ (we neglect chain rupture).

4. The separation δ is governed by the normal pressure of the upper body on the lower one and by the specific force of interaction between the surfaces of bodies 1 and 2. This force is an attraction for δ large, but repulsive forces appear for δ small, so there is a maximum for a gap $\delta°$; Figure 2 shows this force $f(r)$ as a function of the distance r between the bodies, while δ_* corresponds to the minimum possible gap. The attraction at δ large is due to fluctuation fields outside the bodies; theory [4] for large gaps shows that a δ^{-3} law applies, while experiments [5] show that this applies down to gaps of about 0.04 μ.

5. The chains tend to come together when the surface of body 1 is extended along the x axis, so the distance b(Q) between chains obeys

$$db / dQ < 0, \qquad b(0) = b_0 . \tag{1.2}$$

Thus body 1 is represented as a network of force centers with period $2l(Q)$ along the x axis and 2b(Q) along the z axis, one set of centers being at a fixed distance δ from the surface of body 2, while the next set has a distance h(Q) from this surface given by

$$h = \delta + (a^2 - l^2)^{1/2}, \; dh / dQ < 0, \; h(0) = h_0 . \tag{1.3}$$

We assume that there are finite limits b_∞ and h_∞ as $Q \to \infty$.

It is convenient, as in [2], to introduce the periodic functions $\sigma_1(r)$ and $\sigma_2(r)$ whose periods correspond to the periodic structures of bodies 1 and 2 in the direction of motion and which are such that the force of interaction per unit area of body 2 is

$$\mathbf{F}(\mathbf{r}, vt) = \sigma_2(\mathbf{r}) \int \sigma_1(\mathbf{r}' - \mathbf{e}vt) f(|\mathbf{r} - \mathbf{r}'|) dV_1 \tag{1.4}$$

in which the integration is taken over the volume V_1 of body 1; here allowance is made for the relative motion of constant velocity v, e being unit vector along the x axis.

We expand $\mathbf{F}(\mathbf{r}, vt)$ as a Fourier series:

$$\mathbf{F}(\mathbf{r}, vt) = \sum_{m, n} \mathbf{F}_{m, n} \exp\left[i\pi\left(\frac{\mathbf{m}\mathbf{r}}{L} - \frac{nvt}{l}\right)\right]. \tag{1.5}$$

It is obvious that the y projection $F_y(\mathbf{r}, vt) = 0$; it has been shown [2] that the conservation of energy in the absence of motion gives $F^{(x)}_{0,0} \equiv 0$. There is no loss of generality in assuming that $F^{(z)}_{0,0} \equiv 0$.

2. ENERGY DISSIPATION AND THE FORMAL EXPRESSION FOR THE FRICTIONAL FORCE

To determine the energy flux away from the contact surface we must solve for the propagation of elastic waves in the spaces $z > 0$ and $z < 0$ in response to excitation at $z = 0$ by the periodic load of (1.4). For $z < 0$ this reduces to solving the wave equation for $\psi = \operatorname{div} \mathbf{U}$ ($\mathbf{U}$ being the displacement vector), which is easily done by separating the variables [2]. It has been shown that decaying Rayleigh waves are possible for

$$\mathrm{mm} > (nvL / c_1 l)^2, \qquad \mathrm{mm} > (nvL / c_2 l)^2,$$

and undamped sine waves for

$$\mathrm{mm} < (nvL / c_1 l)^2, \qquad \mathrm{mm} < (nvL / c_2 l)^2.$$

Here c_1 and c_2 are the velocities of propagation of longitudinal and transverse waves respectively.

An expression has been derived [2] for the energy flux W_- in the lower half-space on the assumption that $\mathbf{m} \approx 0$:

$$W_- = \sum_{n=1}^{\infty} \{(\rho\mu)^{-1} F^{(x)}_{0,n} F^{(x)}_{0,-n} + [\rho(\lambda + 2\mu)]^{-1} F^{(z)}_{0,n} F^{(z)}_{0,-n}\}. \qquad (2.1)$$

Here ρ is density, λ and μ are Lamé coefficients, and $F^{(x)}_{0,n}$ and $F^{(z)}_{0,n}$ are Fourier coefficients for the projections of $\mathbf{F}(\mathbf{r}, 0)$.

Similarly we can derive W_+, the flux into space $z > 0$. Here the chains of body 1 may be oriented in the direction of motion not only in the surface but also in the volume if the load is sufficiently high. Then there will be several longitudinal and transverse waves that propagate with different velocities, but this is not a vital feature.

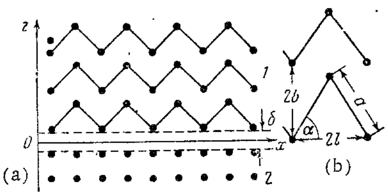

Fig. 1

If again we put $\mathbf{m} \approx 0$, the expression for W_+ is entirely analogous to (2.1). Equating $W_+ + W_-$ to the work of friction per unit time, we get

$$Q = v^{-1} \sum_{n=1}^{\infty} (\omega_1 F^{(x)}_{0,n} F^{(x)}_{0,-n} + \omega_2 F^{(z)}_{0,n} F^{(z)}_{0,-n}). \qquad (2.2)$$

Here ω_1 and ω_2 are quantities dependent on the elastic constants of both bodies; if both are crystalline, the summed expression is independent of Q and v, so we get the falling velocity characteristic of [2].

Above we have followed [2] in assuming $\mathbf{m} \approx 0$. A reasonable physical basis can be given for this assumption: the elastic waves are generated by the periodic structures of bodies 1 and 2 moving with a relative velocity v, as (1.4) shows. However, here we

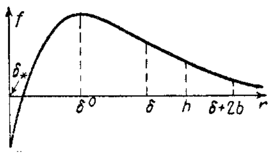

Fig. 2

envisage only the propagation of existing waves, so, if the only waves of importance are those with wavelengths much greater than the repeat distances, we can treat both bodies as continuous media. This corresponds to $\mathbf{m} = 0$ and also to $v \ll \min(c_1, c_2)$.

3. DEPENDENCE ON RELATIVE VELOCITY

Consider the velocity characteristic of Q when the right-hand side of (2.2) depends on Q. For simplicity we consider (2.2) in the linear approximation, i.e., we put

$$d\omega_1 / dQ = 0, \qquad d\omega_2 / dQ = 0.$$

It may be assumed that the energy flux is proportional to the number of force centers per unit of body 1, in which case

$$W_+ + W_- \sim l^{-1}. \qquad (3.1)$$

From symmetry, we may replace the $\mathbf{F}(\mathbf{r}, 0)$ of (1.5) by the force of interaction between the planar net of body 1 and the surface net of body 2. Here we may assume that, on average, $\mathbf{F}^{(x)}(\mathbf{r}, 0) \approx 0$, while $\mathbf{F}^{(z)}(\mathbf{r}, 0)$ is periodic in x and may be prepresented as a sum of terms describing the interaction of the surface of body 2 with planar nets of body 1 at distances of δ, h, $\delta + 2b$, etc. The forces are of short range, so we have from (1.5) for the z-projection of $\mathbf{F}(\mathbf{r}, 0)$

$$F^{(z)}(\mathbf{r}, 0) \approx \sigma_2(x) \sum_k B_k f(h_k), \qquad B_k = C_k l^{-1/2}, \qquad (3.2)$$

in which $h_k = \delta$, h, $\delta + 2b$, etc; the C_k are constants of the order of unity and of the same sign, while $f(\mathbf{r})$ is as in Fig. 2. Substitution of (3.2) into (2.2) then gives

$$W_+ + W_- = Qv = l^{-1} \Big(\sum_{n=1}^{\infty} \sigma_n \sigma_{-n}\Big) \Big(\sum_k C_k f(h_k)\Big)^2. \qquad (3.3)$$

Here σ_n are the Fourier coefficients of $\sigma_2(x)$. Substituting (3.3) into (3.2) and taking only the first three terms in the sum

$$\sum_k C_k f(h_k)$$

(it will be clear that this does not affect the results), we get

$$Q \approx (vl)^{-1} [A_1 f(\delta) + A_2 f(h) + A_3 f(\delta + 2b)]^2 \quad (3.4)$$

in which $A_1 \sim A_2 \sim A_3$ are new constants. Differentiation of (3.4) with respect to Q readily gives

$$\frac{dQ}{dv} = lQ\left\{-v\frac{d(lQ)}{dQ} + 2[A_1 f(\delta) + A_2 f(h) + A_3 f(\delta + 2b)] \times \left[A_2 \frac{df}{dr}\bigg|_{r=h} \frac{dh}{dQ} + A_3 \frac{df}{dr}\bigg|_{r=\delta+2b} \frac{d(2b)}{dQ}\right]\right\}^{-1}. \quad (3.5)$$

Expression (3.5) nowhere becomes zero, provided that none of the derivatives on the right in (3.5) becomes infinite, which is assumed to be so; hence Q(v) has no turning points.

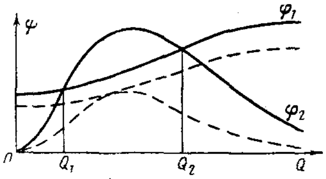

Fig. 3

The actual δ correspond to forces of attraction between the bodies; it is natural to assume that $h > \delta°$, $\delta + 2b > \delta°$. Figure 2 shows that $df/dr < 0$ in this region. From (1.2) and (1.3) we conclude that the terms within the braces in (3.5) have different signs.

Consider the case where the denominator of (3.5) becomes zero. Substitution of (3.1) into (3.5) gives

$$\varphi_1(Q) = [A_1 f(\delta) + A_2 f(h) + A_3 f(\delta + 2b)] \frac{d(lQ)}{dQ} = \varphi_2(Q) = 2Ql\left[A_2 \frac{df}{dr}\bigg|_{r=h} \frac{dh}{dQ} + A_3 \frac{df}{dr}\bigg|_{r=\delta+2b} \frac{d(2b)}{dQ}\right]. \quad (3.6)$$

It is readily seen that the left-hand side takes finite (nonzero) values throughout the range of Q, with a monotonic rise from the value for Q = 0 to that for $Q \to \infty$. The right-hand side of (3.6) tends to zero for $Q \to 0$ and $Q \to \infty$ (we assume that the derivatives of $l - l_\infty$ and $b - b_\infty$ with respect to Q tend to zero more rapidly than as Q^{-1} and $Q \to \infty$). This means that the right side of (3.6) has a maximum for at least one Q. For sufficiently large values of the derivatives on the right in (3.6) this function may exceed in value the left side of (3.6), in which case the curves representing the right and left sides of (3.6) as functions of Q meet at least at the two points Q_1 and Q_2, the derivative of (3.5) becoming infinite at these points (Fig. 3).

Consider these special points in more detail. From (3.4) for $v \to 0$ we get

$$Q \approx (vl_\infty)^{-1} [A_1 f(\delta) + A_2 f(h_\infty) + A_3 f(\delta + 2b_\infty)]^2 = T_0 / v.$$

Similarly for $v \to \infty$ we have

$$Q \approx (vl_0)^{-1} [A_1 f(\delta) + A_2 f(h_0) + A_3 f(\delta + 2b_0)]^2 = T_\infty / v.$$

The derivative of (3.5) is negative in these asymptotic regions, and Q(v) decreases as v increases. The derivative of (3.5) changes sign at Q_1 and Q_2, which corresponds to rise in Q(v) from Q_1 to Q_2 over the range $[v_1, v_2]$. Hence the points (Q_1, v_1) and (Q_2, v_2) in the (Q, v) plane represent cusps in Q = Q(v), while the curve is as in Fig. 4. The first point corresponds to a minimum and the second to a maximum. The presence of a maximum is in good agreement with the experimental evidence [3].

Consider now the general behavior of the curve of Fig. 4 response to change in normal pressure. Increased pressure causes δ, h, $\delta + 2b$, etc. to move to the left in Fig. 2; δ in the region $r < \delta°$ corresponds roughly to an upper bound to the frictional force as a function of pressure, df/dr decreasing in magnitude, and the right side of (3.6) ultimately becomes less than the left for all Q or coincides with it at one point (broken lines in Fig. 3). That is, the region of increasing friction as a function of velocity vanishes as the pressure is raised.

The effects of elasticity are as follows. An inelastic body corresponds in this model to derivatives of h, b, etc. with respect to Q that are small, so the right side of (3.6) becomes less than the left. The right side increases with the elasticity, and for certain critical values of the parameters its curve touches the curve of Fig. 3 representing the left part of (3.6). Above this there appears a region as in Fig. 4.

The limits of applicability of these qualitative results are as follows: The upper bound of the velocity range follows directly from the above:

$$v \ll c$$

in which c is the least of the velocities of elastic waves in bodies 1 and 2. The lower bound to the range of v in which this applies has [2] been discussed for elastic bodies:

$$Q(v) = \frac{\text{const}}{v} < \sum_{n=1}^{\infty} F_{0,n}^{(z)} F_{0,-n}^{(z)}. \quad (3.7)$$

In the present case the lower bound coincides as to order with the solution of (3.7), which gives extremely low velocities, e.g., v of 0.5–1 cm/sec for steel on steel.

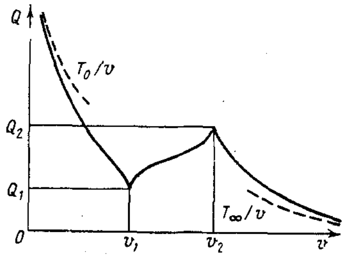

Fig. 4

It is of interest to compare Fig. 4 with the results of [2]. Q is very small for high v (but not ones in excess of c); the tensile stresses in the rubberlike body are small, so the problem becomes that considered in [2]. Hence the asymptote of the curve of Figure 4 for large v is represented by the broken line, T_∞/v. The friction increases at lower speeds, and the stresses extend the polymer chains, which brings the force centers of bodies 1 and 2 closer together. There is thus a rapid rise in the force of interaction in some range $v > v_2$ near v_2. Ultimately the extension of the chains reaches a limit, which leads to fall in the force in range $v_1 < v < v_2$. The chains are maximally extended as v continues to decrease, and the surface layers of body 1 have an oriented structure; neglecting possible chain failure at high Q, we have a situation closely resembling that envisaged in [2]. Hence for $v < v_1$ the Q(v) curve of Figure 4 tends to the asymptote T_0/v (shown by the broken line), with $T_0 \neq T_\infty$, in general.

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